SINTEF

Variational quantum algorithms

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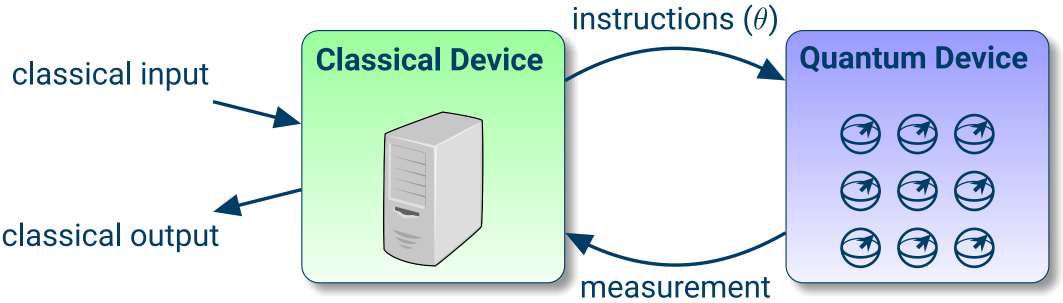
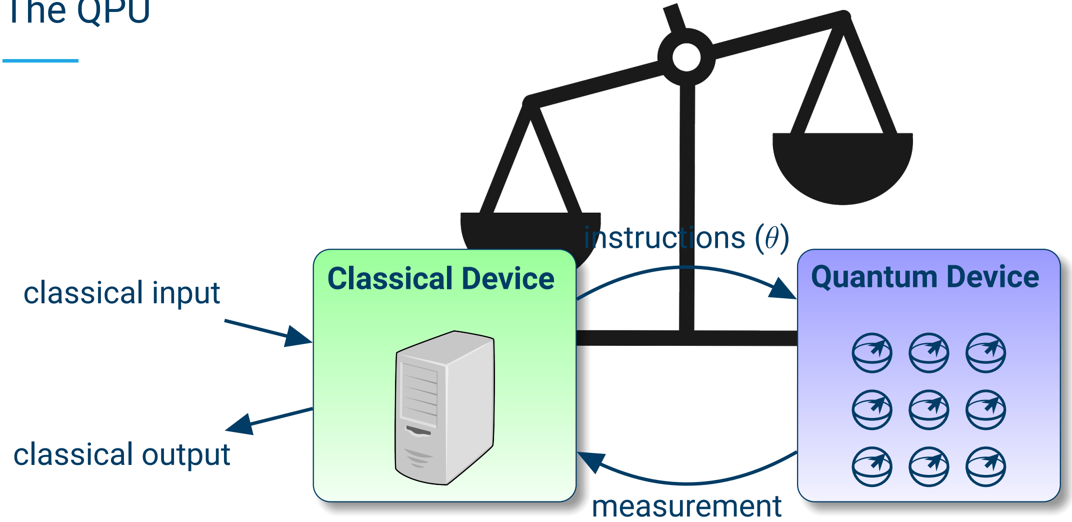
The QPU


instructions (θ)
classical input
Classical Device
Quantum Device
classical output
measurement

The QPU



classical input
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instructions (θ)
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Quantum Device

Variational Principle

Given:

- a state $|\phi\rangle$
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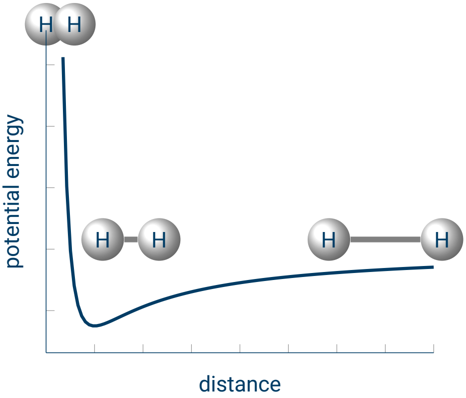
Idea:

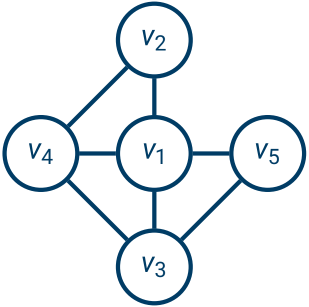
- Encode problem in λ_{min}

Quantum Chemistry



$$H(x) = \sum_{p,q} h_{p,q}(x) a_p^\dagger a_q + \frac{1}{2} \sum_{p,q,r,s} h_{p,q,r,s}(x) a_p^\dagger a_q^\dagger a_r a_s + h_{\text{nuc}} \quad (2)$$

Combinatorial optimization



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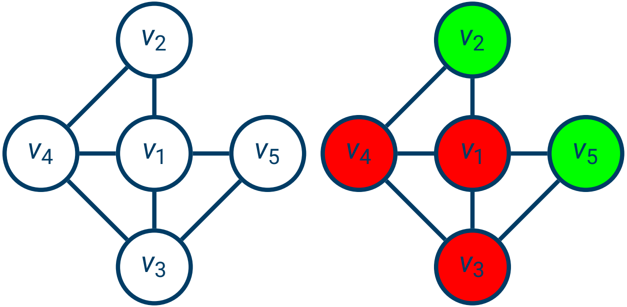

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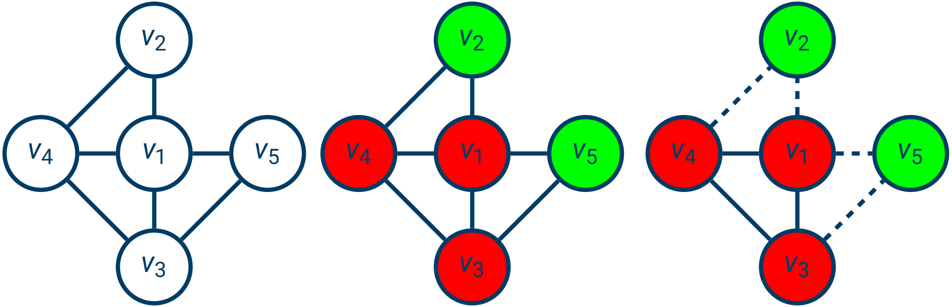

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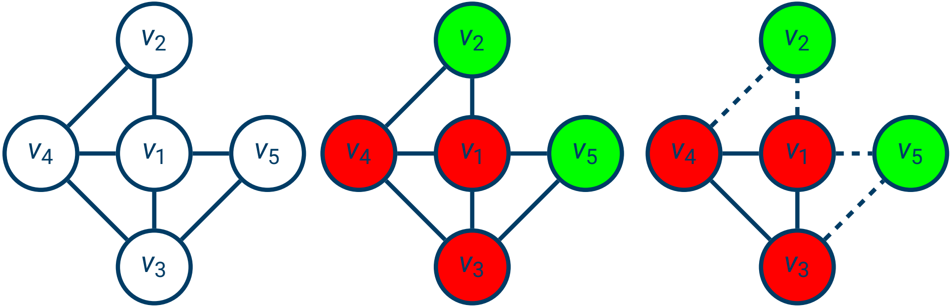


$$H = \sum_{i,j} a_{i,j} Z_i Z_j + \sum_i b_i Z_i \tag{3}$$

Express problem as ground state of Hamiltonian

$$H_{j,k} = \mathrm{diag}\,(+1, -1, -1, +1) = Z_j \otimes Z_k, \quad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{4}$$

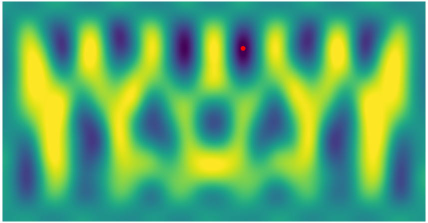
Observe that, e.g.,

- $H_{j,k}\,|00\rangle = +1\,|00\rangle$,
- $H_{j,k}\,|01\rangle = -1\,|01\rangle$,
- $H_{j,k}\,|10\rangle = -1\,|10\rangle$,
- $H_{j,k}\,|11\rangle = +1\,|11\rangle$

Global continuous optimization problem

original problem $\xrightarrow{\textit{Hamiltonian}}$ minimize

$$\text{cost}(\theta) = \langle\phi(\theta)| A |\phi(\theta)\rangle$$



Different ansätze: $|\phi(\theta)\rangle = U(\theta)\,|0\rangle$

Example Ansatz

Using $2p$ parameters $\gamma = \gamma_1, \ldots, \gamma_p$, $\beta = \beta_1, \ldots, \beta_p$, prepare state

$$|\Psi(\gamma, \beta)\rangle = U_{B_p} U_{C_p} \ldots U_{B_1} U_{C_1} |+\rangle^{\otimes n}, \tag{5}$$

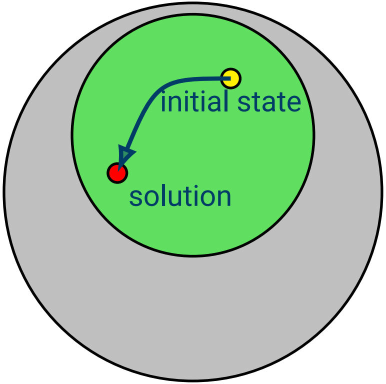
where the operators have the explicit form

$$\begin{aligned} U_{B_l} &= e^{-i\beta_l H_B} = \prod_{j=1}^{n} e^{-i\beta_l \sigma_x^j}, \\ U_{C_l} &= e^{-i\beta_l H_C} = \prod_{(j,k)\in E} e^{-i\gamma_l w_{j,k} H_{i,j}}, \end{aligned} \tag{6}$$

Expressability of Ansatz

Theory

$\mathbb{C}^{2^n}$



Expressability of Ansatz